Cosmic spin and mass evolution of black holes and its impact

Dipanweeta Bhattacharyya
Post Doctoral Fellow
Indian Institute of Science Education and Research (IISER) Mohali

Work done with
Arun Mangalam (IIA Bangalore) and Jasjeet Singh Bagla (IISER Mohali)

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Indian Institute of Astrophysics, Sarjapur Road, Koramangala 2nd Block, Bangalore-560034, India; dipanweeta@iiap.res.in, mangalam@iiap.res.in
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Abstract

We build an evolution model of the central black hole that depends on the processes of gas accretion, the capture of stars, mergers, and electromagnetic torque. In the case of gas accretion in the presence of cooling sources, the flow is momentum driven, after which the black hole reaches a saturated mass; subsequently, it grows only by stellar capture and mergers. We model the evolution of the mass and spin with the initial seed mass and spin in ΛCDM cosmology. For stellar capture, we have assumed a power-law density profile for the stellar cusp in a framework of relativistic loss cone theory that includes the effects of black hole spin, Carter's constant, loss cone angular momentum, and capture radius. Based on this, the predicted capture rates of 10^{-5} to 10^{-6} yr^{-1} are closer to the observed range. We have considered the merger activity to be effective for $z \lesssim 4$, and we self-consistently include the Blandford–Znajek torque. We calculate these effects on the black hole growth individually and in combination, for deriving the evolution. Before saturation, accretion dominates the black hole growth (~95% of the final mass), and subsequently stellar capture and mergers take over with roughly equal contributions. The simulations of the evolution of the $M_{\bullet}$–σ relation using these effects are consistent with available observations. We run our model backward in time and retrodict the parameters at formation. Our model will provide useful inputs for building demographics of the black holes and in formation scenarios involving stellar capture.

Unified Astronomy Thesaurus concepts: Black hole physics (159); Accretion (14); Stellar dynamics (1596); Cosmological evolution (336); Galaxy nuclei (609)

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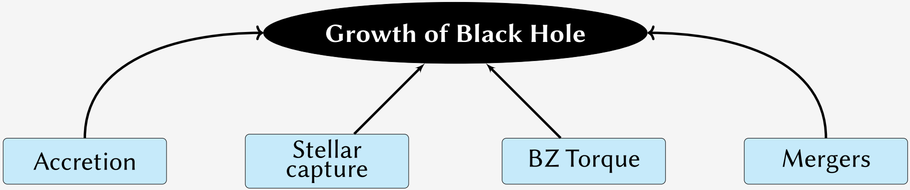

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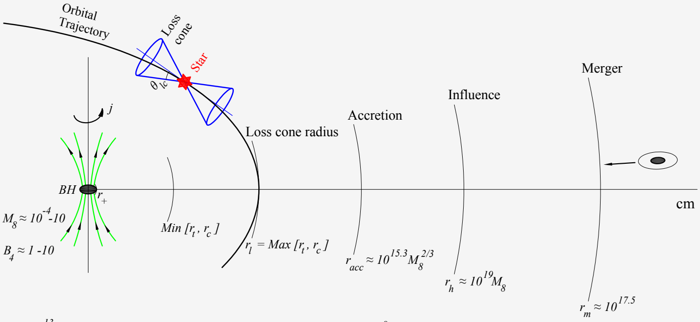
Effects	Region	τ_j	τ_M
Gas accretion	$r_I - r_d$	1 Gyr	1 Gyr
Stellar capture	$r_t - r_h$	-	10 Gyr
Mergers	r_M	10 Gyr	$\sim$ 10 Gyr
BZ Torque	$r_H - r_I$	1 Gyr	-

The domains of the processes



$r_c \approx 10^{13} M_8 \,(1+\sqrt{(1-j^2)}\,)$ cm

$r_t \approx 10^{13.7} M_8^{1/3}$ cm

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- In both the cases the rate of growth of mass of the black hole is proportional to the mass of the black hole. So, here for our calculation we have used :

$$\dot{M}_g = k_1 M_\bullet,$$

where,

$$k_1 = \frac{\eta 4\pi G m_p}{\sigma_e c}$$

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- The black hole growth can occur by both gas accretion till it reaches a saturated mass $M_{\bullet t}$ at a time $t = t_s$. This happens because the outflow velocity exceeds the escape velocity of the medium and the gas is driven away causing the accretion process to stop. The saturated mass is given by $M_{\bullet t} = 9.375 \times 10^6 \sigma_{100}^4 M_\odot$.

Consumption of stars

- For a Kerr black hole, the standard effective potential is written as [Misner et al. (1973); Carter (1968); Frolov & Novikov (1998); Rana & Mangalam (2019a); Rana & Mangalam (2019b)(RM19)]

$$V_{eff}(x, l, j, Q) = -\frac{1}{x} + \frac{l^2 + Q}{2x^2} - \frac{[(l-j)^2 + Q]}{x^3} + \frac{j^2 Q}{2x^4},$$

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- Capture radius (MBSO) $x_c(Q, l, j)$ in units of r_g, is found to be (RM19)

$$x_c^8 - 8x_c^7 - 2j^2 x_c^6 + 16x_c^6 + 2j^2 Q x_c^5 - 8j^2 x_c^5 - 6j^2 Q x_c^4 + j^4 x_c^4 - 2j^4 Q x_c^3 + \\ 8j^2 Q x_c^3 + j^4 Q^2 x_c^2 - 2j^4 Q x_c^2 - 2j^4 Q^2 x_c + j^4 Q^2 = 0.$$

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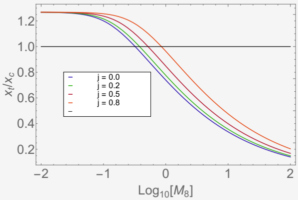
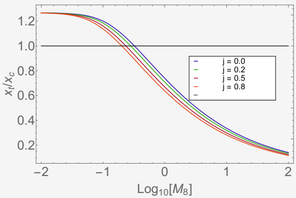
- The loss cone radius $x_\ell \equiv Max[x_t, x_c]$ is given by

$$x_\ell(M_8, j, Q) = r_\ell/r_g = Max[r_t(M_8, j, Q), r_c(j, Q)]/r_g.$$

Critical mass and Loss cone radius





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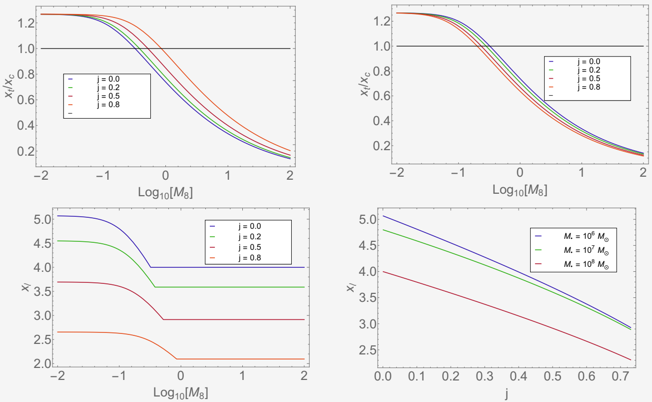


Figure: Ratio of tidal radius to the capture radius ($r_t/r_c = x_t/x_c$) as a function of M_8. We show both prograde (left) and retrograde (right) cases (up) and (down) Loss cone radius ($x_\ell = r_\ell/r_g$) = Max[x_t, x_c] as a function of M_8 (left) and j (right) for prograde case.

Steady loss cone theory

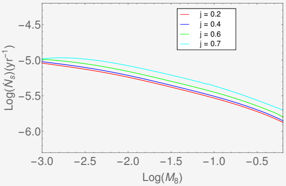
- We started from the basic equation for N_s

$$N_s = 4\pi^2 \int P(E)\,dE \int f_s(E,\mathcal{J})\,d\mathcal{J}^2$$

- We used the effective Kerr potential of black hole for deriving the distribution function of the stars in presence of the total potential of stars and black hole.
- We finally arrive at

$$\frac{d\dot{N}_s}{d\epsilon_s}(M_\bullet, j, k, Q, \epsilon_s, \sigma) = \frac{4\pi^3 L_\ell^2(M_\bullet, j, k, Q)\sigma^5}{G^3 M_\bullet^2 < m_* >} g(\epsilon_s) \frac{\zeta(q_s)}{1 + q_s^{-1}\zeta(q_s)\log(1/R_\ell)}, .$$

- Integrating this expression numerically, we finally find the rate of consumption of stars for the case of the steady loss cone.



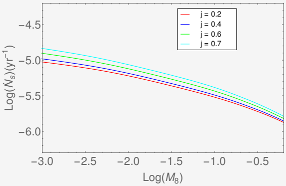


Figure: A plot of $\dot{N}_s$ (M_6) for different values of j when k = 1, prograde (left), -1, retrograde (right) with lower limit of the ϵ_s integration, ϵ_m = -10, γ = 1.1 and σ = 240 km/sec.

Mergers: Mass evolution

- The expression for merger rate is given by Stewart et al(2009) as

$$\frac{dN}{dt} = A_t(z, M)F(m/M),$$

where, m and M are the masses of the smaller and larger merging galaxies with m/M = 0.1 - 0.7.

$$A_t(z, M) = 0.02Gyr^{-1}(1+z)^{2.2}M_{12}^{b},$$

with $b = 0.15$ and M_{12} = M / $10^{12}h^{-1}M_{\odot}$ with $h = 0.7$.

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- Therefore, the rate of mass growth due to merger is given as

$$\frac{dM}{dt} = A_t M \int_q^1 F(q)q dq,$$

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- We write the integral part of the equation as $n(q)$ and therefore the final equation becomes,

$$\dot{M}_m = \frac{1}{f_h}\frac{dM_\bullet}{dt} = 0.02(1+z(t))^{2.2}\left[\frac{0.7M_5}{10^7 f_h}\right]^{0.15} n(q)\frac{M_5}{f_h},$$

where, $f_h = M_\bullet/M$ and the equation is expressed in units of $10^5 M_{\odot}$ / Gyr, M_5 is mass of the SMBH in units of $10^5 M_{\odot}$.

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- Therefore, we neglect the contribution of the major mergers and consider only the minor mergers and the spin up of the black hole occurs only due to the accretion process.
- We use the expression from Gammie et. al (2004) for including the effect of minor mergers in spin evolution of the black hole as

$$\left.\frac{d\log j}{d\log M_\bullet}\right|_m = -\frac{7}{3} + \frac{9q}{\sqrt{2}j^2},$$

BZ effect

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- The spin down due to BZ torque is given by the expression (Mangalam et al. 2009)

$$\frac{dj}{dt} = r_+^3(j)j\frac{\mathcal{G}_0}{\mathcal{J}_0},$$

where, $r_+(j) = 1 + \sqrt{1 - j^2}$, BZ Toque,

$$\mathcal{G}_0 = \frac{m^3}{8}B_\perp^2 f = 4 \times 10^{46} fB_4 M_8^3 (erg),$$

and the angular momentum budget is

$$\mathcal{J}_0 = cM_\bullet m = 9 \times 10^{64} M_8^2 (g\ cm^2\ s^{-1}).$$

Evolution of the black hole

Spin evolution equation

$$\frac{dj}{d\tau} = \frac{\dot{\mu}_g}{\mu_\bullet}\Big(l_I(j) - 2\epsilon_I(j)j\Big) + \frac{\dot{\mu}_*}{\mu_\bullet}\Big(l_*(j) - 2\epsilon(j)j\Big) + \dot{\mu}_m \cdot \frac{j}{\mu_\bullet}\Big(-\frac{7}{3} + \frac{9q}{\sqrt{2}j^2}\Big) + \frac{4}{9} \times 10^{-5} f_{BZ} B_4 \mu_\bullet M_{s5} x_H^3(j) j.$$

Mass evolution equation

$$\frac{d\mu_\bullet}{d\tau} = \epsilon_I(j)\dot{\mu}_g + \epsilon(j)\dot{\mu}_* + \dot{\mu}_m.$$

Our model

Case 1 : $z_s < 4$

$$z : z_f \to 4 : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet g} + \dot{M}_{\bullet *} \\ \dot{j} = \dot{j}_g + \dot{j}_{BZ}. \end{cases}$$

$$z : 4 \to z_s : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet g} + \dot{M}_{\bullet *} + \dot{M}_{\bullet m} \\ \dot{j} = \dot{j}_g + \dot{j}_{BZ} + \dot{j}_m. \end{cases}$$

$$z : z_s \to 0 : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet *} + \dot{M}_{\bullet m} \\ \dot{j} = \dot{j}_{BZ} + \dot{j}_m. \end{cases}$$

Case 2 : $z_s > 4$

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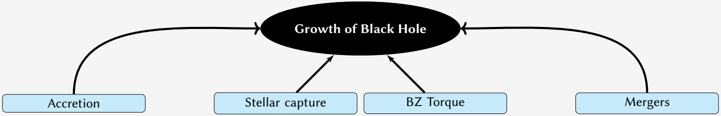
$$z : z_s \to 4 : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet *} \\ \dot{j} = \dot{j}_{BZ}. \end{cases}$$

$$z : 4 \to 0 : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet *} + \dot{M}_{\bullet m} \\ \dot{j} = \dot{j}_{BZ} + \dot{j}_m. \end{cases}$$



Our model

Case 1 : $z_s < 4$

$$z : z_f \to 4 : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet g} + \dot{M}_{\bullet *} \\ \dot{j} = \dot{j}_g + \dot{j}_{BZ}. \end{cases}$$

$$z : 4 \to z_s : \begin{cases} \dot{M}_\bullet = \dot{M}_{\bullet g} + \dot{M}_{\bullet *} + \dot{M}_{\bullet m} \\ \dot{j} = \dot{j}_g + \dot{j}_{BZ} + \dot{j}_m. \end{cases}$$

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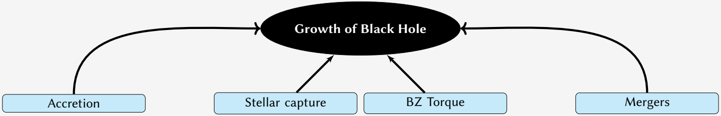
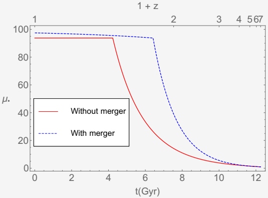


ΛCDM Model of Cosmology

$$t(z) = \frac{1}{H_0} \int_{1/(1+z_f)}^{1/(1+z)} da \frac{1}{\sqrt{\Omega_m a^{-1} + \Omega_\Lambda a^2}}, = t_z(z) - t_z(z_f),$$

$$t_z(z) = \frac{1}{H_0} \frac{2}{3} \frac{1}{\sqrt{1-\Omega_m}} \log \left[\sqrt{1-\Omega_m} \sqrt{\Omega_m - \frac{\Omega_m - 1}{(1+z)^3}} - (\Omega_m - 1) \left(\frac{1}{1+z} \right)^{\frac{3}{2}} \right]$$

Evolution with and without mergers



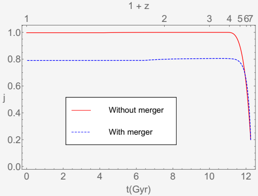


Figure: Evolution of $\mu_\bullet(t)$ (a) and (b) $j(t)$ of the black hole are shown without and without the effect of mergers for the canonical case.

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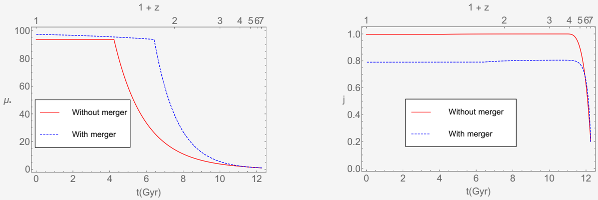

Figure: Evolution of $\mu_\bullet(t)$ (a) and (b) $j(t)$ of the black hole are shown without and without the effect of mergers for the canonical case.

- It is clearly seen from mass evolution that in presence of the mergers, the black hole reaches the saturation mass earlier due to the higher mass growth rate and that the final mass attained is higher.
- As we consider the merger activity to be effective from $z \lesssim 4$, we see that the two curves start deviating from each other after $z \gtrsim 4$.
- The saturated or the final spins are different for the two cases due to the minor mergers which cause the spin down of the black holes.

Application 1: Evolution of the $M_\bullet - \sigma$ relation

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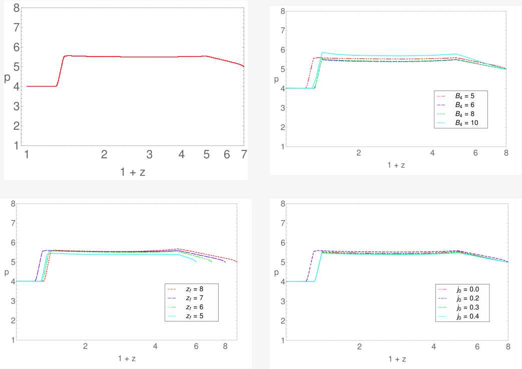

Figure: The evolution of the index $p(z)$ for $\gamma = 1.1$, $M_s = 10^4 M_\odot$.

#	Galaxy	$M_\bullet$ (in $10^7 M_\odot$)	σ (km/sec)	z
1	NGC 3379	13.6	230	0.00304 ± 0.00001
2	NGC 3377	2.60	217	0.00222 ± 0.00001
3	NGC 4486	188	433	0.00428 ± 0.00002
4	NGC 4551	3.77	218	0.00392 ± 0.00002
5	NGC 4472	117	542	0.00327 ± 0.00002
6	NGC 3115	17.0	230	0.00221 ± 0.00001
7	NGC 4467	0.493	77	0.00475 ± 0.00004
8	NGC 4365	67.7	453	0.00415 ± 0.00002
9	NGC 4636	58.0	251	0.00313 ± 0.00001
10	NGC 4889	299	467	0.02167 ± 0.00004
11	NGC 4464	1.12	112	0.00415 ± 0.00001
12	NGC 4697	20.76	215	0.00414 ± 0.00001

Table: Data from BM18a used for matching our results with observations.

#	References	p	k_0
1	Ferrarese & Merritt (2000)	4.8	0.5
2	Gebhardt et al. (2000)	3.75	0.9
3	Merritt & Ferrarese (2001)	4.72	0.5
4	Ferrarese (2002)	4.58	0.7
5	Tremaine et al. (2002)	4.02	0.83
6	Ferrarese & Ford (2005)	4.86	0.57
7	Gultekin et al. (2009)	4.24	0.7
8	Kormendy & Ho (2013)	4.38	1.48
9	McConnell & Ma (2013)	5.64	0.42
10	Debattista et al. (2013)	4.06	0.97
11	Batiste et al. (2017)	4.76	1.69
12	Sahu et al. (2019)	6.10	0.27

Table: The historical determinations of the Slopes and Constant of the $M_\bullet - \sigma$ relation in units of M_7 and σ_{100}.

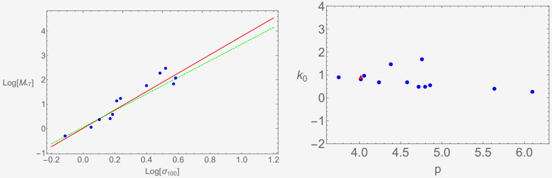

Figure: $\log(M_{\bullet 7})$ vs $\log(\sigma_{100})$ for [$z = 0.003$, (red) and $z = 0.23$, (green)] from our model compared with the data from BM18a for the 12 elliptical galaxies (z = 0.004 - 0.002), the index $k_0(z)$ for γ = 1.1, M_s = $10^4 M_\odot$ for the canonical case at $z = 0$ compared with values from literature.

Application 2: Seed black hole formation through stellar capture by heavy stellar mass black hole seeds

- Using our relativistic steady loss cone theory, the mass growth rate due to stellar capture alone can be approximated to be

$$\dot{M}_{\bullet *} = 5 \times 10^{-6} M_6^{-0.33} M_{\odot}, yr^{-1}, \;\; For\, \sigma = 200\; kms^{-1}, \gamma = 1.1.$$

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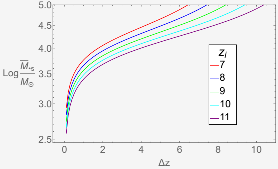

Figure: The mass growth as a function of change in redshift where it is seen that $10^4 M_\odot$ seed is obtained for $\{z_i, z_f\}$ = $\{\{11, 7.01\}, \{10, 6.63\}, \{9, 6.21\}, \{8, 5.74\}, \{7, 5.23\}\}$.

- For σ = 200 km sec^{-1} and γ = 1.1, $\bar{M}_{\bullet s}^{1.33} - \bar{M}_{\bullet *}^{1.33} \simeq \bar{M}_{\bullet s}^{1.33} = 6.35 \times 10^5 \Delta t$;
- $\Delta t = t(z_f) - t(z_i)$; $\Delta z = z_i - z_f$, where the masses are in units of $M_\odot$, and $\Delta t = t(z_f) - t(z_i)$ is in units of Gyr.
- Therefore, it is seen that SMBH seeds of $10^4 M_\odot$ can be formed in 10^7–10^8 years depending on the initial redshift range, z_i = 7–10.

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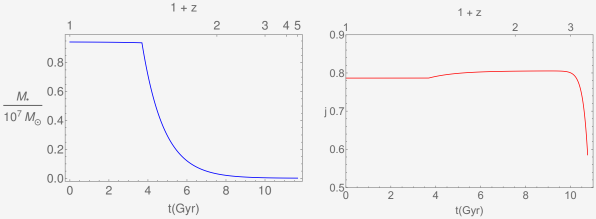

Figure: (a) $M_\bullet(t)$ and (b) $j(t)$ for the complete model starting from final mass $\mu_{\bullet 5}$ = 100.

- From the mass and spin values for the quasars listed in Campitiello et al. (2019), (as determined through KERRBB and SLIMBH models), the following input sets of $\{\eta, j_f\} = \{\{1, 0.7\}, \{0.1, 0.7\}, \{1, 0.45\}, \{0.1, 0.45\}\}$ are suggested. They also calculated $M_\bullet$ for $j_f = \{0, 1\}$. We have taken the final mass to be $M_\bullet \simeq 10^9 M_\odot$ at $z \simeq 7$ and evolved our model backwards the for different sets of $\{\eta, j_f\}$ to find the initial seed masses at $z_f = 20$.

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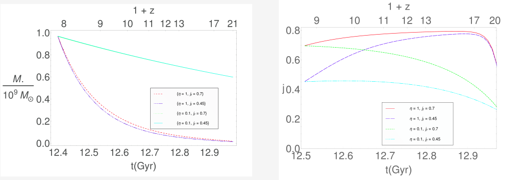

Figure: $M_\bullet(t)$ and $j(t)$ for different combinations of η and j_f at $z \simeq 7$, till $z = 20$ for final mass at $z \simeq 7$, $M_f \simeq 10^9 M_\odot$ (a, b).

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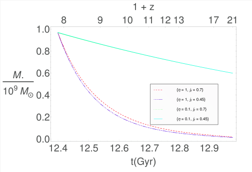

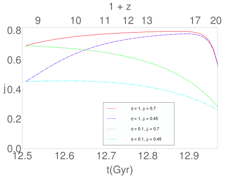

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- For $\eta = 1$, the seed mass is also lower by a factor of nearly 60 as compared with the case of $\eta = 0.1$; this is expected due to the difference in accretion rate. The j_f values does not make much difference to $M_\bullet(t)$ when η is fixed.
- For the case of spin evolution, when $\eta = 1$, the j increases and then decreases, but for $\eta = 0.1$, it continues to decrease. For higher η, the spin reaches maximum value rapidly and then it reduces due to the presence of BZ torque and minor mergers but, when $\eta = 0.1$, the mass growth is slower, so it does not reach the maximum spin within the short time of less than a Gyr.
- It seems that a seed of nearly $M_s = 10^7 M_\odot$ is possible at $z = 20$ only if $\eta = 1$.

Where can we find the merged Black hole in BH-BH mergers? D Bhattacharyya & J S Bagla (2022)(in prep)

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    - BH misses the bulge and becomes a halo object. It can be expected to gradually slow down due to dynamical friction during disk crossings and gradually get trapped in the disk and then sink towards the bulge.

Summary

- We have included relativistic effects in the process of tidal and direct capture and built a semianalytic self-consistent evolution model of the black hole.
- We have explored the roles and phases of importance of each of the growth channels. Though the contributions from stellar capture ($\sim 3\%$) and mergers ($\sim 2\%$) in mass growth are small compared to accretion ($\sim 95\%$), these two play major roles after the saturation. BZ torque contributes only to the spin-down of the black hole (for $B_4 = 10$, the spin-down is $\sim$3% from the max value attained owing to accretion). Mergers and the BZ process are necessary; otherwise, the black holes will be spinning maximally.
- We illustrated the effect of saturation on the evolution of the $M_\bullet(z) = K_0(z)\sigma^{p(z)}$ relation.
- By running the models backward in time, we retrodict the formation parameters of seed black holes. This will enable us to discriminate among models of black hole formation.
- Stellar capture can be considered as a viable process for formation of SMBH seeds, as this dominates the accretion process when $M_\bullet \leq 2 \times 10^4 M_\odot$.
- We expect our transparent and detailed formulation in a fully relativistic framework to be useful for future simulational studies.
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